Mathematical Model of Bridge-Linked Photovoltaic Arrays Operating Under Irregular Conditions

Modelo Matemático de Arreglos Fotovoltaicos en Puente-Vinculado Operando bajo Condiciones Irregulares

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Abstract

This paper presents a mathematical procedure to model a photovoltaic array (N rows and M columns) in bridge-linked configuration operating under regular and irregular conditions. The proposed procedure uses the ideal single-diode model representation for each photovoltaic module and the Shockley equation to represent each bypass diode. To pose the system of NxM non-linear equations required to obtain the voltages of each module of the array, the proposed model apply the Kirchhoff current law to each internal node of the array and the Kirchhoff voltage law to: each loop created in the upper part of each connection of two consecutive strings and each string with respect to the output voltage of the array. Moreover, the model provide a procedure to obtain the Jacobian matrix to reduce the solution time of the equation system. The circuital model of two arrays (small: 3x3 and medium: 20x3) were implemented in Simulink to validate the proposed model. The accuracy and improved calculation speed of the proposed model allow its use to perform energetic evaluations of bridge-linked arrays or its comparison with other typical array configurations, which can be useful in the designing of photovoltaic plants.

Keywords

Photovoltaic array model; bridge-linked; irregular conditions; partial shadowing; mismatching.

Resumen

Este artículo presenta un procedimiento matemático para modelar un arreglo fotovoltaico (N filas y M columnas) en configuración puente-vinculado operando en condiciones regulares o irregulares. Dicho procedimiento usa el modelo ideal de un diodo para representar cada módulo fotovoltaico y la ecuación de Shockley para representar cada diodo de desviación. Para plantear el sistema de NxM ecuaciones no lineales requerido para obtener los voltajes de cada módulo, el modelo propuesto aplica la ley de corrientes de Kirchhoff a cada nodo interno del arreglo y la ley de voltajes de Kirchhoff a: cada lazo creado en la parte superior de una conexión entre dos columnas consecutivas y a cada columna con respecto al voltaje del arreglo. Además, el modelo provee un procedimiento para obtener la matriz Jacobiana para reducir el tiempo de solución del sistema de ecuaciones. El modelo circuital de dos arreglos (pequeño: 3x3 y mediano: 20x3) fue implementado en Simulink para validar el modelo. La exactitud y mejora en la velocidad de cálculo del modelo propuesto permite su uso para realizar evaluaciones energéticas de arreglos en puente vinculado o su comparación con otras configuraciones típicas, lo cual puede ser útil en el diseño de plantas fotovoltaica.

Palabras clave

Modelo de arreglo fotovoltaico; puente-vinculado; condiciones irregulares; sombreados parciales; discrepancia.

1.   INTRODUCTION

Photovoltaic (PV) systems are one of the most important renewable energy source due to their advantages: modularity, zero emissions, free energy source, availability of the energy source, among others. All those advantages, along with polices and subsides implemented for different countries, have promote the PV marked in the recent years (IEA, 2010). The largest part of such installed systems are grid connected applications (IEA, 2010); in such applications it is very common the irradiance and temperature conditions of the PV modules are different due to the partial shades produced by the surrounding objects (trees, buildings, antennas, dust, etc.) and clouds, as well as differences in the parameters of the PV modules that compose the PV array.

Such irregular operation conditions are known as mismatching conditions and produce multiple maximums in the Power vs. Voltage (P-V) curves of the array due to the activation and deactivation of the diodes connected in parallel to each PV module (bypass diodes). The presence of multiple maximums (Local Maximum Power Points - LMPP) in the P-V curves may produces a significant reduction in the maximum available power in the PV array, since the control system that searches the operating point that provides the maximum power may be trapped in an LMPP instead of the global maximum (Global Maximum Power Point - GMPP).

The Series Parallel (SP) and Total Cross-Tied (TCT) configurations are array configurations most widely adopted in the literature for both, modeling PV arrays (Bastidas et al., 2013; Hiren Patel & Agarwal, 2008a; Ramos-Paja et al., 2012) and proposing new MPPT techniques (H. Patel & Agarwal, 2008b; Velasco-Quesada et al., 2009); however, there are other possible configurations, like Bridge Linked (BL), that provide alternative interconnection between the modules and may present some advantages for a particular application.

The BL arrays are a tradeoff between SP and TCT (see Fig. 1) since they implement approximately half of the additional connections between two consecutive strings with respect to the TCT configuration. The additional connections start after the second module between the first and the second string and are imple-

mented every two modules. Between the second and third string the additional connections start after the first module and are implemented, again, every two modules. The additional connections of the further strings are connected following the same pattern (Wang & Hsu, 2011).
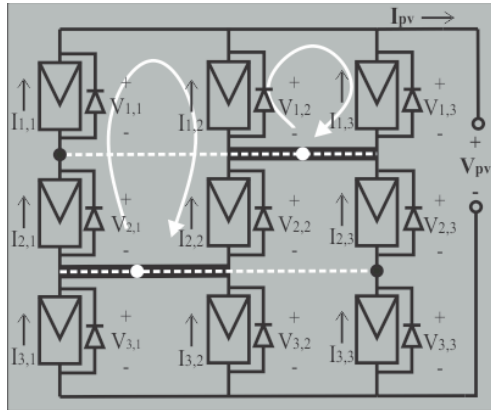

Fig. 1. PV arrays of 3 columns and 3 rows in SP (connections in thin continuous lines), TCT (SP connections plus white dashed lines) and BL (SP connections plus thick black lines) configurations. Source: Authors

The authors, so far, have found few information about modeling and simulation of BL arrays in mismatching conditions. Only in (Wang & Hsu, 2011) the authors pose the system of equations to be solved for an specific BL array (9x4) to validate a piece-wise linearized model proposed in such a paper. Nevertheless, in (Wang & Hsu, 2011) there is not any generalized procedure to or method model a BL array. Moreover, in (Picault et al., 2010) the authors present some general indications to be considered in the definition of the non-linear equation system when there is a connections between two columns of a PV array in SP configuration; nonetheless, any general procedure to model BL arrays is presented.

This paper presents a mathematical model to calculate the current and power of BL arrays considering a simplified representation of the PV modules and the exponential model of the bypass diodes with the purpose of: reducing the calculation time, calculate the Maximum Power Point (MPP), and remove the requirement of

circuital simulators in energy evaluation analysis; in that way, the proposed model is suitable to be implemented in standard programming languages such as Matlab script and C.

2. METHODOLOGY

2.1 Model of a Single PV Module

The ideal single diode model of a PV module with a bypass diode connected in anti-parallel to its output is presented in Fig. 2.
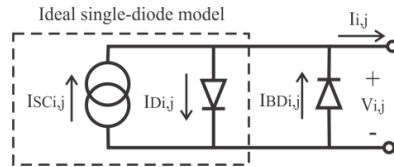

Fig. 2. Ideal single-diode model with bypass diode. Source: Authors

Using the Kirchhoff current law in the circuit shown in Fig. 2, it is possible to write an explicit equation of the output current of the module at the position **i,j** of the array ($I_{i,j}$) as a function of its output voltage ($V_{i,j}$) as shown in (1). The parameters $A_{i,j}$, $B_{i,j}$ and $I_{SCi,j}$ can be evaluated based on the datasheet information, for a given irradiance ($G_{PVi,j}$) and temperature ($T_{PVi,j}$) conditions (Petrone & Ramos-Paja, 2011), by using (2)-(5). In such equations $I_{SCSTCi,j}$ and $V_{OCSTCi,j}$ are the short circuit current and open-circuit voltage in Standard Test Conditions (STC), respectively; the temperature and irradiance of the PV module in STC are $T_{STCi,j}$ and $G_{STCi,j}$, respectively. $B_{STCi,j}$ is the value of the parameter $B_{i,j}$ in STC, $I_{MPPi,j}$ and $V_{MPPi,j}$ are the current and voltage of the module in the MPP, respectively, for a given weather conditions. The coefficients $\alpha_{ISCi,j}$ and $\alpha_{VOCi,j}$ are the current and voltage temperature coefficients. Finally, the parameters of the bypass diodes are represented by $A_{BDi,j}$ and $B_{BDi,j}$, which represent the inverse saturation current and the inverse of the thermal voltage, respectively.

$$I_{i,j} = I_{SCi,j} - A_{i,j} \cdot \exp\left(B_{i,j} \cdot V_{i,j}\right) + A_{BDi,j} \cdot \exp\left(-B_{BDi,j} \cdot V_{i,j}\right) \tag{1}$$

$$I_{SCi,j} = I_{SCSTCi,j} \cdot \frac{G_{PVi,j}}{G_{STCi,j}} \cdot \left(1 + \alpha_{ISCi,j} \cdot \left(T_{PVi,j} - T_{STCi,j} \right) \right) \tag{2}$$

$$B_{i,j} = \frac{B_{STCi,j}}{1 + \alpha_{VOCi,j} \cdot \left(T_{PVi,j} - T_{STCi,j} \right)} \tag{3}$$

$$B_{STCi,j} = \frac{\ln \left(1 - \left(I_{MPPi,j}/I_{scSTCi,j} \right) \right)}{V_{MPPi,j} - V_{OCSTCi,j}} \tag{4}$$

$$A_{i,j} = I_{scSTCi,j} \cdot \exp\left(-B_{STCi,j} \cdot V_{OCSTCi,j} \right) \tag{5}$$

2.2 Modeling of a BL PV Array

The model of a BL array can be formalized by defining the system of non-linear equations to be solved for a given operation conditions. Such a system of equations is written to calculate the voltage of each module in the array, then (1) is used to calculate the current of each module; in that way, all the electric variables of the array will be known.

In a PV array composed by **N** modules in each string and **M** strings connected in parallel it is necessary to write a system of **NxM** non-linear equations to solve the **NxM** unknown voltages of the array. Such **NxM** non-linear equations are obtained by applying the Kirchhoff current (KCL) and voltage (KVL) laws to the array.

An array of **NxM** has **N-1** nodes between the modules in each string, where the node 1 is between the module 1 and module 2 and node **N-1** is between modules **N-1** and **N**. For any node that connects two consecutive strings the KCL is described by (6), where $I_{a,b}(V_{a,b})$ represents the current function of the module **a** in the string **b** as a function of its voltage $(V_{a,b})$ as shown in (1). **a** is the number of the node and **b** is the string at the left of the node. In the odd strings **a** only takes even values, while in the even strings **a** only takes odd values. Such a convention is assumed to avoid the repetition of nodes in the creation of the non-linear equation system.

$$I_{a,b}(V_{a,b}) + I_{a,b+1}(V_{a,b+1}) - \left(I_{a+1,b}(V_{a+1,b}) + I_{a+1,b+1}(V_{a+1,b+1})\right) = 0 \qquad (6)$$

In the first and last strings there are nodes formed between two consecutive modules connected in series. In such cases the application of KCL results in (7), where **c** only takes odd number for the first string. For the last string **c** only takes odd numbers if **M** is even and vice versa.

$$I_{c,d}(V_{c,d}) - I_{c+1,d}(V_{c+1,d}) = 0 \qquad (7)$$

The connections between the strings create loops composed by the four modules located at the upper part of the string. Applying the KVL to the loop created by the node **a** which connects the strings **b** and **b+1** (8) is obtained.

$$V_{a-1,b} + V_{a,b} - \left(V_{a-1,b+1} + V_{a,b+1}\right) = 0 \qquad (8)$$

The first node that connects an even string with an odd string (e.g. connection between string 2 and string 3 in Fig. 1) creates a simple loop of two modules connected in parallel; therefore the KVL is described by (9), where **f** only can take even values.

$$V_{1,f} - V_{1,f+1} = 0 \qquad (9)$$

Each string of the BL array is connected in parallel to power converter (e.g. inverter), which sets the voltage of the array (V_{pv}) to track the GMPP of the system or to guarantee a fixed array voltage; therefore, the voltage of each string fulfills (10), where **k** goes from 1 to **M**.

$$V_{pv} - \sum_{i=1}^{N} V_{i,k} = 0 \ \ \forall \, k \in [1, M] \qquad (10)$$

Considering the general equations presented in (6)-(10) it is possible to pose a system of **NxM** non-linear equations required to find the voltages of the array. On the other hand, the definition of the Jacobian matrix it is required to reduce the solution time of

the system. To write such a matrix the derivative of the current of a module with respect to its voltage is required (11). Moreover, the unknowns of the system (voltages) are organized according to the vector $\mathbf{V_{vec}}$ presented in (12).

$$\frac{dI_{i,j}}{dV_{i,j}} = -B_{i,j} \cdot A_{i,j} \cdot \exp(B_{i,j} \cdot V_{i,j}) - B_{BDi,j} \cdot A_{BDi,j} \cdot \exp(-B_{BDi,j} \cdot V_{i,j}) \qquad (11)$$

$$V_{vec} = \begin{bmatrix} V_{1,1} \ V_{2,1} \ \cdots \ V_{N,1} \ V_{1,2} \ V_{2,2} \ \cdots \ V_{N,2} \ \cdots \ V_{1,M} \ V_{2,M} \ \cdots \ V_{N,M} \end{bmatrix} \qquad (12)$$

The procedure used to define the system of non-linear equations and the Jacobian matrix is presented in Fig. 3. Such a procedure obtains a vector with the system of non-linear equations ($\mathbf{F}$) and the Jacobian matrix ($\mathbf{J}$) by applying the KCL and KVL along the PV array. Then $\mathbf{F}$ and $\mathbf{J}$ are used by a numerical method to find the voltages of all the modules in the array.

3.   RESULTS AND DISCUSSION

For the simulations presented below the ERDM 85 PV module was considered as a reference. The main datasheet information of such a module is: V_{OCSTC}=21.78 V, V_{MPPSTC}=17.95 V, I_{SCSTC}=5.13 A, I_{MPPSTC}=4.8 A, α_{ISC}=0.020 %/K, α_{VOC}=-0.34 %/K. The P600J diode was selected as a reference for the bypass diode. Such a diode has a voltage drop of 0.11 V for a forward current of 8 A and an inverse saturation current of 1 μA.

The model parameters of the modules were calculated by using (4) and (5) considering a temperature of 25 ᵒC (A=7.5992e-7 A and B=0.7220 1/V) while the model parameters of the bypass diode were calculated from the datasheet information (A_{BD}=1 μA and B_{BD}=144.927 1/V).

3.1 Simulation of a Small and Medium BL Array

The circuital models of a small BL array of three 3x3 (see Fig. 1) and a medium array (20x3) were implemented in Simulink to compare the accuracy and of the proposed model. The current vs.

voltage (I-V) and P-V curves of the small and medium PV arrays are presented in Fig. 4a and Fig. 4b, respectively.
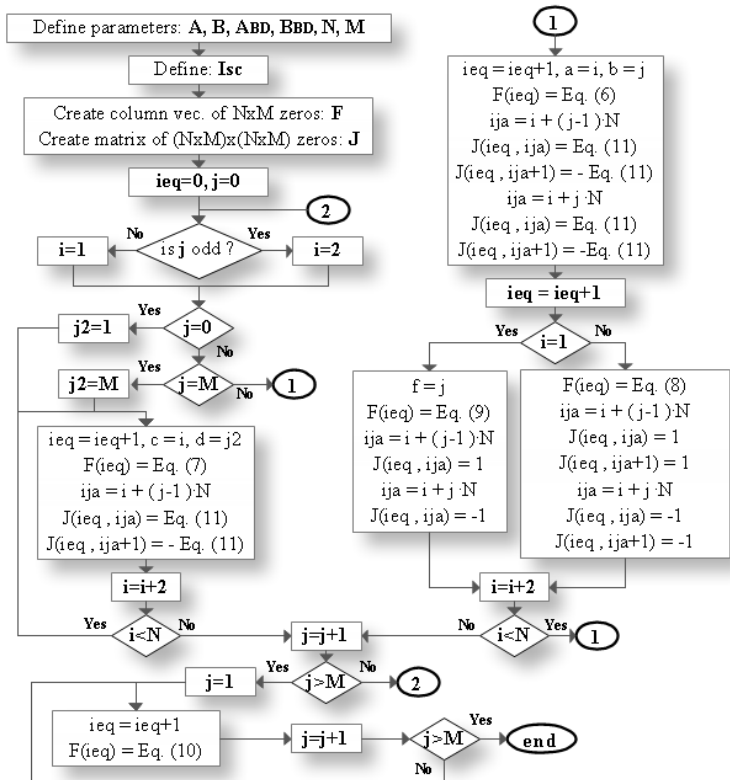

Fig. 3. Algorithm to write non-linear equation system and Jacobian matrix.
Source: Authors

For the 3x3 array two conditions were simulated. The first one with the system operating under STC conditions and the second conditions with the array operating under mismatching conditions. For the mismatching conditions the short-circuit currents of the nine modules, as a percentage of I_{SCSTC}, are: $I_{SC1,1}$=90 %, $I_{SC2,1}$=60 %, $I_{SC3,1}$= 20 %, $I_{SC1,2}$=80 %, $I_{SC2,2}$=100 %, $I_{SC3,2}$= 10%, $I_{SC1,3}$=50 %, $I_{SC2,3}$=40 % and $I_{SC3,3}$=30 %. In both cases the agreement between the circuital and proposed model is reflected in the

low Normalized Sum Squared Errors (NSSE), which are 0.0023 % for STC and 0.0367 % mismatching conditions.

The characteristic curves presented in Fig. 4b show the agreement between the circuital implementation and the proposed model in a medium PV array operating under mismatching conditions. In this case, the short-circuit currents of the modules were generated randomly to simulate a complex mismatching pattern. Again, the low NSSE (0.0226 %) confirms the quality of the proposed model for medium PV arrays. For this array the simulation time consumed by Simulink to calculate 4000 points was 1371,7 s, while the time consumed by the proposed model was 94.4 s (14.5 times faster), which demonstrate the usefulness of the proposed in the reduction of the simulation time of an array with respect to a circuital implementation.

3.2 Energy Estimation in a Medium BL Array

To predict the potential energy that can be produced by a PV array in a given period of time it is necessary to calculate the P-V curve of the array for each value of irradiance and calculate the GMPP of the curve. Then, the values of GMPP are integrated along the time to obtain the potential energy that can be produced by the array. In this example the energy is calculated for an irradiance record of one day with samples every minute (Fig. 5a) considering the 20x3 array presented in the previous section.

Two different simulations were performed to illustrate the effect of inverting the order of the modules in the second column of the array. The GMPP vs. time plots for both cases are presented in Fig. 5b, where the black line corresponds to the array with the same configuration used for the previous section (config. 1) and the gray line corresponds to the array with the second column inverted (config. 2). It is observed that the change of the order reduces in 6 % the energy obtained from the array in one day, that is why the tools that allow to simulate a PV array in different configurations are important for the designing of a PV plant.
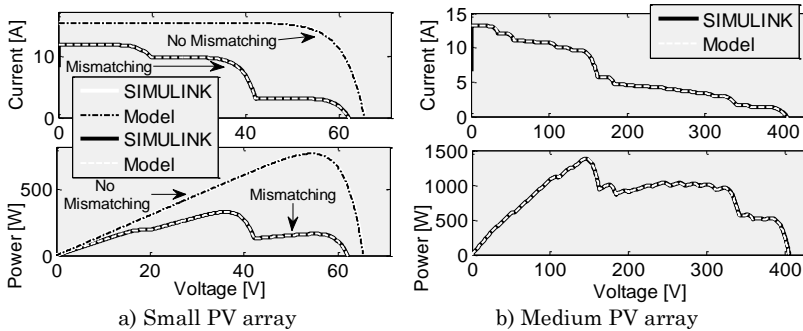
a) Small PV array   b) Medium PV array
Fig. 4. Characteristic curves of a small and medium PV array. Source: Authors

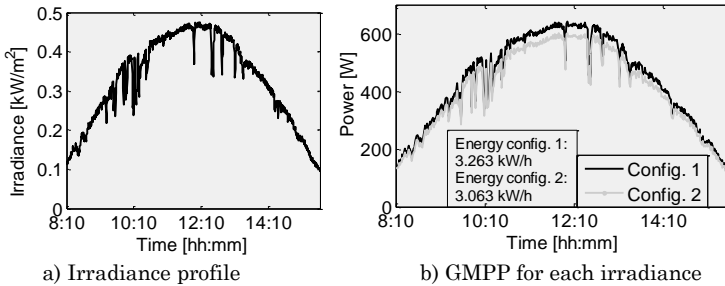a) Irradiance profile   b) GMPP for each irradiance
Fig. 5. Irradiance profile and GMPP for a 20x3 BL array in two configurations.
Source: Authors

4. CONCLUSIONS

The model presented in this paper is able to simulate a PV array in BL configuration composed by N modules in each string and M strings connected in parallel. The accuracy and improved calculation time, with respect to a circuital implementation, as well as the simplicity of the model implementation in a standard programming language (e.g. Matlab, C, etc.) show that the proposed model is suitable for its use in different applications. For example, it can be used in the designing of PV plants to determine the best distribution of the array modules according to the particular mismatching conditions. Moreover, the model can be used in algorithms that search the best configuration of a BL array to increase the maximum power available in the PV array as shown in

(Bastidas-Rodriguez et al., 2013; Velasco-Quesada et al., 2009) for SP and TCT applications, respectively. In this paper the ideal single diode-model was used to represent each module to simplify the calculations; however, the algorithm presented in Fig. 3 can be used for a more accurate model like the one presented in (Petrone, Spagnuolo, & Vitelli, 2007). In such a case the equations (1) and (11) should be replaced by the equivalent explicit functions of the module voltage presented by the authors in the mentioned paper.

5.   ACKNOWLEDGEMENTS

6.   REFERENCIAS

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