

## **Position Fuzzy Control for a Two-Axis Shaking Table based on Slider-Crank Mechanism**

### ***Control de Posición Fuzzy para una Mesa Vibratoria de Dos Ejes Basada en un Mecanismo Biela-Manivela***

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**Abstract**

Different mechanisms have been designed to generate vibratory motion to test the evaluation of seismic control systems to be used in structural buildings. These systems are called "shaking-tables" and they are usually designed with linear actuators which facilitate the implementation of classical control systems for its proper operation. This paper presents a position fuzzy control system designed to control the displacement behavior of earthquakes on the shaking-table based on a slider-crank mechanism. The results show repeatability greater than 97%, adequate to the validation of anti-seismic controllers on small-scale models.

**Keywords**

Shaking table; seismic simulator; slider-crank mechanism; fuzzy control; intraclass correlation coefficient ICC.

**Resumen**

Diferentes mecanismos se han diseñado para generar movimientos vibratorios que ayuden a la evaluación de sistemas de control antisísmicos, para ser usados en edificaciones civiles. Estos sistemas denominados "mesas vibratorias" se diseñan generalmente con actuadores lineales los cuales facilitan la implementación de sistemas de control clásicos para su correcto funcionamiento. Este trabajo presenta un sistema de control fuzzy, orientado a controlar el comportamiento de desplazamiento de movimientos telúricos sobre una mesa vibratoria basada en un mecanismo biela-manivela; los resultados presentan una repetitividad superior al 97%, adecuada para la validación de controladores antisísmicos en modelos a pequeña escala.

**Palabras clave**

Mesa Vibratoria; simulador sísmico; mecanismo biela-manivela; control Fuzzy; coeficiente de correlación intraclass ICC.

## 1. INTRODUCTION

A shaking-table is a system used to create agitation movements to be applied in mechanics vibration test or physics models. These models are used to analyze the performance and resistance to similar events (Seki, Iwasaki, & Hirai, 2010; Seki, Iwasaki, & Kawafuku, 2009; Yao, Fu, Hu, & Liu, 2010). In civil engineering they are used to make seismic test on scale models like tall buildings or long bridges, through simulation of earthquakes intended to reproduce the acceleration behavior of an earthquake (Seki, Iwasaki, & Kawafuku, 2009). To perform legitimate experiments, it is necessary to achieve a high degree of reproducibility and to take into account the dynamic behavior of shaking-table to apply the appropriate input signals to the system in order to generate the desired vibrations in the output (Seki, Iwasaki, Kawafuku, Hirai, & Yasuda, 2009; Seki, Iwasaki, & Kawafuku, 2009).

Due to the fact that most of the shaking-tables that exist are designed to generate a profile of acceleration with lineal displacement actuators, the difference in this work is that the shaking-table uses the angular displacement from two induction motors to create linear displacement through a slider-crank mechanism in two axis. This movement is made by the coupling of motors to a chain and gearbox system to the slider-crank mechanism.

Having in mind that the displacement of the shaking-table is generated by an induction motor, the signals to control it are applied to each frequency inverter of motor through the analog option by controlling a digital potentiometer. Accordingly, when the mechanic system is coupled with the electronic control system, the entire system is a machine with high non-linear performance, and as a consequence the mathematical model be complex and with high uncertainty.

On the other hand, due to the fact that earthquakes are stochastic processes, their random features allow the definition the maximum and minimum acceleration limits over a ground surface (Ceccarelli, Pugliese, Lanni, & Mendes Carvalho, 1999). That's the reason the test made over shaking-tables to the structure analysis is based on the reproduction of random earthquakes with known

maximum peaks of acceleration (Chen & Zhang, 2008; Murota, Feng, & Liu, 2006).

The implementation of a Fuzzy control system is an adequate technique to replicate earthquake signals, due to its good performance in systems in which the model is unknown. They allow to obtain a high repeatability, which is very important to evaluate new seismic control systems on scale buildings over the shaking-table. In this particular case, the designed Fuzzy control system is a Sugeno type, with error signal and rate of change of error like input signals and five subsets to each signal, and the controller is implemented on the PIC32 Ethernet Starter Kit from Microchip®.

## 2. IDENTIFICATION OF SHAKING-TABLE

In Fig. 1 the slider-crank mechanism used in the shaking-table is shown from a superior view. In this figure it can be seen that when the motor is turned on, the table moves forward and backward and then it performs a translation in the range of +6 cm to -6 cm that describes a sine trajectory in time, which is the reason a region of lineal behavior was selected.

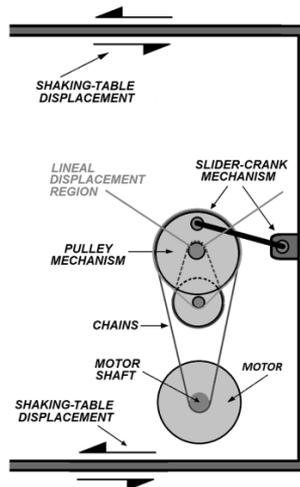


Fig. 1. Slider-Crank mechanism used in shaking-table axis

On the other hand, the use of the Fuzzy control system is not necessarily an approximation to the shaking-table model, so it was calculated by parametric identification techniques to simulate the system control and validate with the real system. These models, from X axis (North-South) and Y axis (East-West) of the shaking-table, are presented in (1) and (2), and they have a one-step prediction of 91.8 % and 91.9 % respectively.

$$G_X = \frac{e^{-0.101 s}(120.4 s + 13.6)}{2.84 * 10^{-5} s^4 + 0.01261 s^3 + 1.403 s^2 + s} \quad (1)$$

$$G_Y = \frac{e^{-0.0773 s}(2.802 s + 77.71)}{0.0008106 s^3 + 0.05694 s^2 + s} \quad (2)$$

To validate the error in the identified model with the real response, we used the Normalized Root Mean Square Error NRMSE given by (3) (Frýza & Hanus, 2003), obtaining errors of 0.0852 and 0.0672 for X axis and Y axis respectively. In this equation the term  $x_i$  is the simulated output and  $y_i$  is the real output of the plant respectively.

$$NRMSE = \sqrt{\frac{\sum_{i=1}^n (x_i - y_i)^2}{\sum_{i=1}^n (x_i)^2}} \quad (3)$$

### 3. FUZZY CONTROL SYSTEM DESIGN

The Fuzzy control system is implemented over a hardware device and a controller based on Sugeno Theory was selected due to its computational efficiency (Ross, 2004) (Jang, Gulley, & Math-Works, 1998). Additionally, the control surface is softer, which benefits the plant reactions by the control system.

The linguistic variables selected to the controller are the error and the error rate signals, both from displacement measurements, and five fuzzy sets were defined to subdivide the range of linguistic variables, with triangular or trapezoidal shape. This membership functions are selected because they are symmetric, a necessary feature to apply the Weighted Average – WA technique in the

Fuzzy Sugeno Controller (Ibrahim, 2003). Fig. 2 shown the final Fuzzy sets, to error and error rate, obtained by tuning, which are distributed on the range from -5 mm up +5 mm. This range is adequate to the control system because the shaking table only supports low frequency earthquakes, so the maximum error rate is 100 millimeters per second, and the sample frequency in this work is 200 Hz.

In Fig. 2 left, the Fuzzy sets are named large negative error (LNE), medium negative error (MNE), zero error (ZE), medium positive error (MPE), and large positive error (LPE); in Fig. 2 right, the Fuzzy sets are called large negative error rate (LNER), medium negative error rate (MNER), zero error rate (ZER), medium positive error rate (MPER), and large positive error rate (LPER). Both are in millimeters.

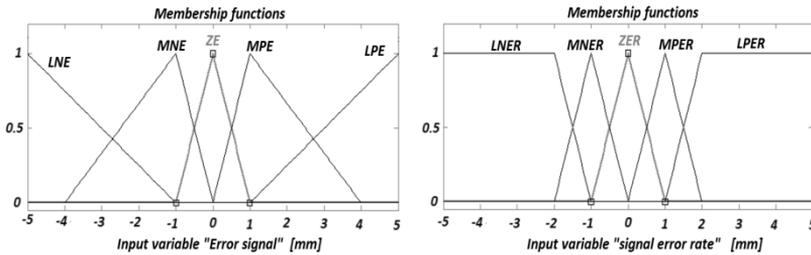


Fig. 2. Fuzzy sets to input error left, and Fuzzy sets to input error rate right

The defuzzification process is based on the Weighted Average technique of all the output rules, and it is calculated by (4) (Jang et al., 1998) (Tanaka & Wang, 2001).

$$Output = \frac{\sum_{i=1}^N \omega_i z_i}{\sum_{i=1}^N \omega_i} \tag{4}$$

Where  $\omega_i$  is the rule weight,  $N$  is the number of rules, that in this case is 25, and  $z_i$  is the output function, which in Sugeno topology is as in (5).

$$z = a \cdot error + b \cdot error\_rate + c \tag{5}$$

Where  $a$  and  $b$  are weight constants to the error and error rate values, and  $c$  is a given offset to the particular output; the parameters  $a$ ,  $b$  and  $c$  are obtained by tuning the controller. From (4) and (5) the control surface for X-axis and Y-axis is obtained, as it is shown in Fig. 3, where the output control corresponds to the digital value that must have digital potentiometer to modify the motor's frequency.

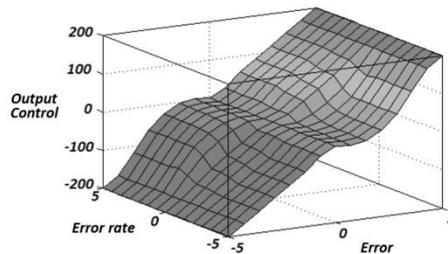


Fig. 3. Control surface of Y-axis

#### 4. SYSTEM CONTROL IMPLEMENTATION

The total system has a software component, made in MATLAB®, and a hardware component, implemented over the PIC32 Ethernet Starter Kit from Microchip®. In the software the seismic signals are uploaded, obtained by the Center of Engineering Strong Motion Data (CESMD), given in acceleration units [cm/s<sup>2</sup>], which are converted to displacement by double integration. A base line error is defined and it is eliminated by signal processing. The displacement signal is used to generate the earthquake in the shaking-table, and this treatment is shown in Fig. 4.

Although the signal showed in Fig. 4 corresponds to the Quetame (Cundinamarca, Colombia) earthquake that happened in May 2008 (INGEOMINAS, 2008; RNAC, 2010). This register wasn't used in these experiments, and the Imperial Valley, CA earthquake of October 15 1979 was used, because it has low frequency components, ideals for experiments.

To begin the earthquake test, in the Matlab® Guide the seismic signal is loaded and pre-processed, then it's transmitted to the hardware by a data packet up to 36.000 bytes that contains the X-

axis and the Y-axis information. This data packet is reordered in the hardware, and it is used to generate the earthquake in two directions, using two encoders with 100 ppr of resolution to sense the displacement and calculated the error signals. With these errors the adequate output for the digital potentiometers is obtained to adjust the motor frequency in the inverters. Finally, the displacement signals are saved in the hardware and later transmitted to the software to be processed and displayed in the graphic interface.

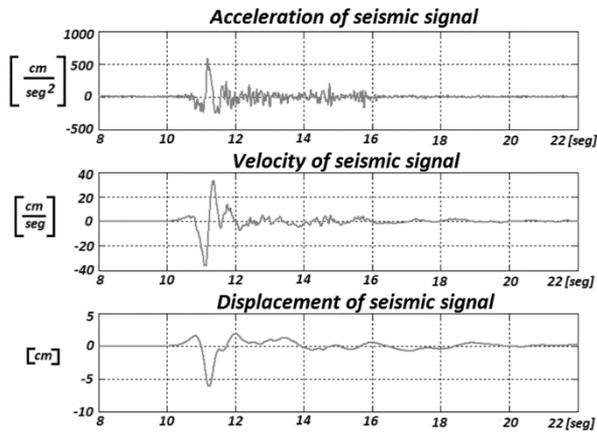


Fig. 4. North-South component of a seismic register converted to velocity and displacement

The encoders used to calculate the errors are installed outside of the shaking-table, through a rack and pinion mechanism as it's shown in Fig. 5.

## 5. RESULTS

Once the X-axis and Y-axis controllers are tuned by simulation, they are implemented in the PIC32 hardware device. The obtained results are shown in Fig. 6, which confirms that the plant models obtained by identification works properly, and it has a good approximation to the real output. In this test, the real output

was obtained without load. In order to validate the test repeatability, tests were made for three different cases, without load and with 20 kg and 30 kg of load. The results are shown in Fig. 7.

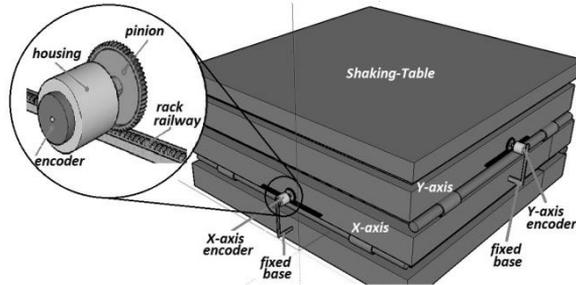


Fig. 5. Location of encoders on Shaking-Table

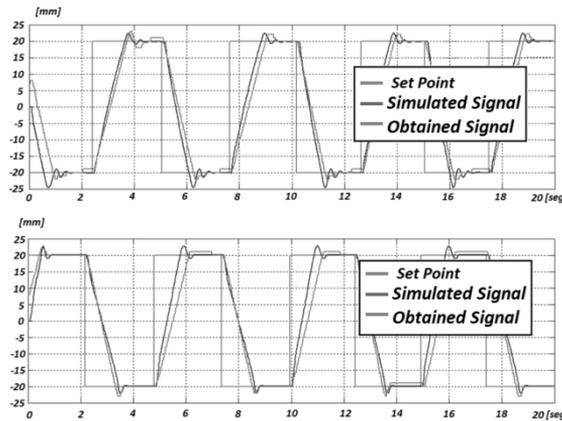


Fig. 6. Comparison of simulated and obtained signals. X-axes and Y-axes

In Fig. 6 and Fig. 7 the result of NRMSE are shown, with an average NRMSE of 0.0606 for X-axis and 0.0704 for Y-axis in Fig. 6, and an average NRMSE of 0.0224 for X-axis and 0.0175 for Y-axis in the repeatability test of Fig. 7.

To compare the results of the Fuzzy Controller we use two coefficients, the coefficient of determination  $R^2$  and the intraclass correlation coefficient ICC, specifically the two-way random effect model. The  $R^2$ , given in percentage, is used to evaluate the amount of variability between the desired signal and the obtained signal;

the ICC is used to estimate the repeatability of experiments because this allows to the systematic bias on T test. The result of ICC is like the Pearson coefficient of correlation, it has a value from 0 to 1 where a number close to 1 indicates a good repeatability. The results obtained in these tests were over 97 % of approximation, by the analysis of  $R^2$  and ICC, and the cumulated NRMSE on the entire signal was below 0.16 for X-axis and below 0.12 for Y-axis. The coefficient of determination  $R^2$  is presented in (6) (Montgomery & Runger, 2003) and the intraclass correlation coefficient ICC in (7) (McGraw & Wong, 1996; Weir, 2005).

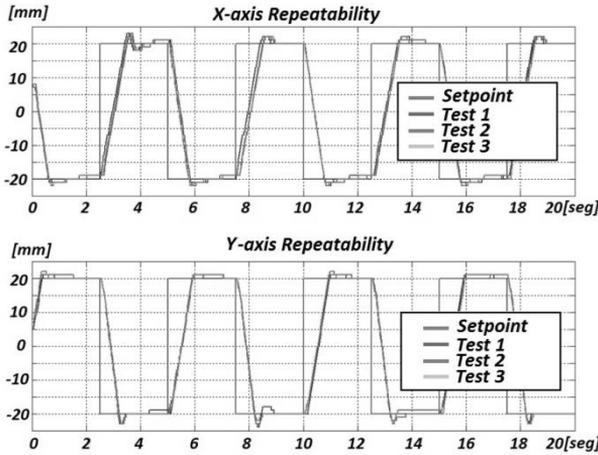


Fig. 7. Repeatability test of Fuzzy control, X-axis and Y-axis

$$R^2 = 1 - \frac{\sum_{i=1}^n (x_i - y_i)^2}{\sum_{i=1}^n (x_i - \mu_{x_i})^2} = 1 - \frac{\text{var}(e)}{\text{var}(x_i)} \quad (6)$$

$$ICC(A, 1) = \frac{MS_R - MS_E}{MS_R + (k - 1)MS_E + \frac{k}{n}(MS_C - MS_E)} \quad (7)$$

In (7),  $MS_R$  is the mean square between rows test,  $MS_C$  is the mean square between columns test and  $MS_E$  is the error between tests,  $n$  is the length of data and  $k$  is the total number of test. The final result is a number that compares all the tests made. Table 1, shows the NRMSE, the  $R^2$  and the ICC of X-axis and Y-axis re-

spectively, for 10 test of the shaking-table with a load given by a scale building of 20 kg; this analysis was made in displacement domain.

Table 1. X and Y Axis displacements results

Test	Seismic signal X-axis			Seismic Signal Y-axis		
	NRMSE	R <sup>2</sup> [%]	ICC(A,1)	NRMSE	R <sup>2</sup> [%]	ICC(A,1)
1	0.15513	97.616		0.11341	98.715	
2	0.14686	97.881		0.11319	98.719	
3	0.1426	97.988		0.11552	98.666	
4	0.13543	98.187		0.10865	98.82	
5	0.1423	98.002		0.10745	98.848	
6	0.14539	97.91	0.99949	0.10931	98.811	0.99966
7	0.14181	98.017		0.13112	98.306	
8	0.13569	98.177		0.11067	98.777	
9	0.14728	97.862		0.1057	98.892	
10	0.1351	98.196		0.10879	98.82	

## 6. CONCLUSIONS

The implemented displacement control system works properly, with high similitude and repeatability of signals, greater than 97 % for both axes. However, the current resolution of the control system is one millimeter, which forces the system to make an approximation of the displacement signal. This introduces high frequency noise components, which deteriorates the seismic signals to emulate, and as a consequence the error is increased.

To reduce this, it is recommended to increase the resolution by the introduction of a gear box to the encoder or to replace the encoders with other whose resolution is higher, because that the current encoder has a resolution of 100 pulses per revolution.

Another fundamental aspect that involves the behavior of this shaking-table is its pulley mechanism, because when it is using chains like the transmission medium, there is a dead space, which introduces a dead time to the transfer functions of X-axis and Y-axis. Nevertheless, this problem cannot be eliminated because it is inherent to the shaking-table, so any control system must deal with it.

Additionally, due to the limitations of the entire mechanism, this particular shaking-table can only emulate earthquakes with low frequency components, fewer than 4 Hz, like the earthquake used in this work. As a consequence, new control systems based in velocity or acceleration are to be implemented, so to corroborate the mechanics capabilities and limitations of the shaking-table, in order to create the seismic laboratory at the University.

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