

## **Energy Prediction in Urban Photovoltaic Systems**

### ***Predicción de Energía en Sistemas Fotovoltaicos Urbanos***

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**Abstract**

This paper proposes a new method to accurately estimate the power and energy production in urban photovoltaic (PV) systems, which are commonly covered by shades affecting its performance. The solution is based on an efficient algorithm designed to compute, in short time, an accurate model accounting for the shades impact. In such a way, the proposed approach improves classical solutions by significantly reducing the processing time to simulate long periods, e.g. months and years, but without introducing sensible errors. Therefore, this approach is suitable to estimate the production of PV systems for economical analyses such as the return-of-invested time calculation, but also to accurately design PV installations by selecting the right number of photovoltaic modules to supply the required load power.

**Keywords**

Urban installations; photovoltaic system; power and energy estimation; fast processing; high precision.

**Resumen**

Este artículo propone un nuevo método para estimar la potencia y energía producida por sistemas fotovoltaicos urbanos, los cuales son comúnmente cubiertos por sombras que afectan su desempeño. La solución se basa en un algoritmo para procesar, rápidamente, un modelo preciso que considera el efecto de las sombras. Esta solución provee un mejor desempeño en comparación con aproximaciones clásicas, ya que reduce significativamente el tiempo de cálculo sin introducir errores sensibles, permitiendo la simulación de largos periodos de operación, e.g. meses y años. Por lo tanto, esta solución es apropiada para realizar estimaciones de energía orientadas a análisis económicos, e.g. cálculo del tiempo de retorno de la inversión, así como para soportar el diseño de instalaciones fotovoltaicas, permitiendo el cálculo preciso del número de módulos requeridos para suplir el perfil de carga.

**Palabras clave**

Instalaciones urbanas; sistemas fotovoltaicos; estimación de potencia y energía; procesamiento rápido; alta precisión.



ples of such a technique are given in (Petroni et al., 2007) and (Petroni & Ramos, 2011), where two different models are proposed to provide different compromises between accuracy and simulation speed.

Taking into account that commercial modules are composed by several PV cells connected in series and protected by a bypass diode, which avoids module destruction by overheating conditions in reverse operation (Silvestre et al., 2009), the PV system simulation becomes a non-trivial task: complex cell models provide high accuracy, as the model introduced in (Petroni et al., 2007), but the required computational effort generates long simulation times, which makes impossible to estimate monthly or yearly performances to provide a long-term analysis. On the contrary, the use of simplified models significantly reduces the simulation times, as the model presented in (Petroni & Ramos, 2011) to estimate long-term performances, but it introduces errors that could lead to wrong designs or non-profitable decisions.

Such conflictive objectives, i.e. high accuracy and fast processing, are addressed in this paper by introducing a novel approach aimed at providing accurate estimations with low computational efforts, i.e. short simulation times. The proposed solution is based on an efficient algorithm designed to compute a complex cell model but simplifying the bypass diode model. The new approach is intended for urban PV installations, which commonly consists of several series-connected PV modules, named PV string, managed by a single PV inverter as in Fig. 1: the minimum number of modules in the string is defined by the minimum voltage required by the PV inverter to operate. The inverter tracks the optimal string voltage to maximize the PV power production, delivering such a power to the grid. Finally, each string is protected with a blocking diode  $D_{bk}$  from destructive negative current flows.

## 2. MODELING THE PV MODULE

Since a PV module consists in series-connected cells in parallel with a bypass diode, two models must be defined: the cells model and the diodes model. In literature, the series-connected cells are

commonly modeled using the single diode model depicted at the left of Fig. 2 (Petrone et al., 2007): it consists of a current source modeling the photo-induced current, a diode  $D_J$  modeling the p-n junction within the cells, and two resistances  $R_h$  and  $R_s$  modeling the saturation and ohmic effects. A simplified version of such a model, depicted at the right of Fig. 2 (Petrone & Ramos, 2011), disregards the contribution of both resistances.

The photo-induced current  $i_{ph}$  is proportional to the effective irradiance reaching the PV module, it including the reduction caused by shades. The  $D_J$  current  $i_d$  is calculated as in (1) (Petrone et al., 2007), while the module current and voltage are calculated in (2) from Kirchhoff laws. In (1),  $i_{od}$  and  $v_t$  represent the saturation current and thermal voltage of  $D_J$ , respectively. Finally, the model parameters ( $i_{od}$ ,  $v_t$ ,  $R_h$  and  $R_s$ ) are calculated following the procedure in (Eicker, 2003) for the five parameters model (including  $i_{ph}$ ).

$$i_d = i_{od}[\exp(v_d/v_t) - 1] \quad , \quad i_h = v_d/R_h \tag{1}$$

$$i_{pv} = i_{ph} - i_d - i_h \quad , \quad v_{pv} = v_d - i_{pv} \cdot R_h \tag{2}$$

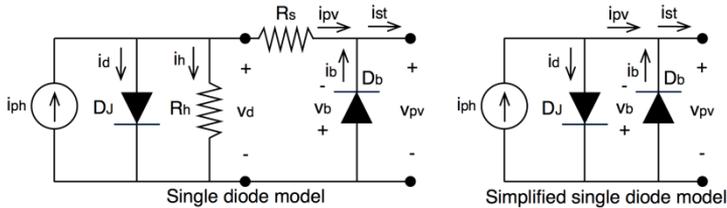


Fig. 2. Single diode model: complete (left) and simplified (right) versions.  
Source: Authors

In the simplified model  $v_{pv} = v_d$ . Moreover, its parameters ( $i_{od}$  and  $v_t$ ) are calculated following the procedure in (Eicker, 2003) for the three parameters model (also including  $i_{ph}$ ). The main differences between both models, apart from their complexity, concern the parameters validity range: the complete model parameters are valid for the whole irradiance range ( $[0, 1000]$  W/m<sup>2</sup> in earth), while the simplified model parameters generate errors when the irradiance diverges from the value used in the parameterization.

A similar approach was followed to model the bypass diode: some papers adopt the Schottky equation to provide high accuracy,

while other papers adopt the ideal switch model to reduce complexity. In such a way, the model proposed in (Petrone *et al.*, 2007), named Lambert-W model, adopts the complete single diode (SD) model and the Schottky equation (left circuit in Fig. 2) to represent the PV modules, while the model proposed in (Petrone & Ramos, 2011), named Inflection Points model, adopts the simplified single diode (SSD) model and the ideal switch equivalent (right circuit in Fig. 2) to represent the PV modules. Both models provide different compromises between accuracy and speed: Lambert-W model provides high accuracy, but its long simulation times makes impossible to evaluate monthly or yearly profiles. In contrast, the Inflection Points model provides short simulation times to evaluate long power profiles, but it introduces errors that could lead to wrong decisions.

To provide a tradeoff between both approaches, this paper proposes to represent the PV cells with the SD model and to represent the bypass diodes with a modified switch: the bypass diode is closed when its voltage is higher than a real diode threshold voltage  $v_b$ , otherwise the diode is open. Fig. 3 shows the current-voltage (I-V) characteristics of the proposed, Lambert-W and Inflection Points models, where the improvement over the ideal switch approach is evident ( $v_b = 0.2$  V). Taking into account that the Schottky equation is an exponential expression similar to (1), the proposed model provides a satisfactory accuracy with a small complexity increment.

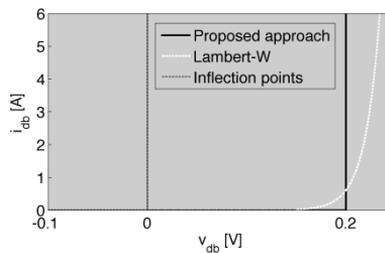


Fig. 3. Comparison of the bypass diodes models. Source: Authors

Similarly, Fig. 4 shows the comparison between the three modules models. At the left, the I-V curves show that the three models accurately describe the short-circuit current ( $I_{sc}$ ), open-

circuit voltage ( $V_{oc}$ ) and maximum power point (MPP). Such a figure also put in evidence the high accuracy provided by the proposed approach, in comparison with the Inflection Points model, for the reproduction of the PV module behavior as power source (first quadrant: positive PV current and voltage).

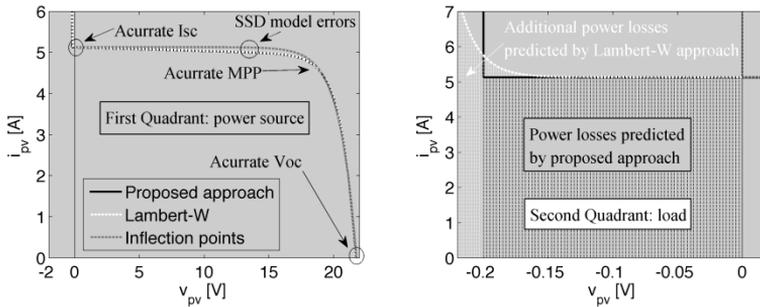


Fig. 4. Comparison of the PV modules models. Source: Authors

The right part of Fig. 4 presents the models behavior at the second quadrant (positive current and negative voltage), in which the PV module operated as a load. Such results show the improvement provided by the proposed model over the Inflection Points approach: since the diode voltage is negative, it forces the module to dissipate power, which is modeled by the  $v_b$  voltage. Moreover, taking into account that the string current imposes the module current, the diode current will be always lower than the maximum  $i_{ph}$  value. In the simulation presented in Fig. 4  $i_{ph} = 5.13$  A, therefore the difference between the proposed and Lambert-W models is negligible.

### 3. MODELING THE PV STRING

As depicted in Fig. 1, a PV string is formed by several (N) modules in series with a blocking diode  $D_{bk}$ . Taking into account that, in general, each PV module could exhibit a particular irradiance, different from the one of the other modules, the bypass diodes become active when its voltage is higher than the threshold voltage  $v_b$  as in Fig. 3. Therefore, the modules operate at the sec-

ond quadrant (as a load,  $p_{pv} < 0$ ) for PV voltages within  $-v_b < v_{pv} < 0$ . Such a condition is considered by both the Lambert-W and proposed approaches; instead, the Inflection Points approach consider the PV module inactive ( $p_{pv} = 0$ ) for  $v_{pv} < 0$  as depicted at the right of Fig. 4.

To estimate the power production of a PV string, the model in (Petrone et al., 2007) evaluates all possible string voltages to find the MPP power for each irradiance condition. Such a procedure permits to estimate the maximum power available for a given irradiance profile of a geographical area. But, to find the MPP power it is required to calculate the string current for the evaluated  $v_{st}$ , which in turn requires solving the  $N+1$  non-linear equations given in (3)-(4), where  $1 \leq k \leq N-1$  and  $(v_{t,bk}, i_{o,bk})$  belong to the blocking diode model. Moreover, since solving (3) uses (1) and (2), each component of (3) becomes an implicit equation that requires the specialized Lambert-W function to find the solution (Petrone et al., 2007), which strongly increases the computational effort. Hence, since the Lambert-W function must be used  $N$  times in each  $v_{st}$  evaluation, the simulation time is very long.

$$i_{st} = i_{pv,k} + i_{b,k} = i_{pv,k+1} + i_{b,k+1} \quad (3)$$

$$v_{st} = \sum_{k=1}^N v_{pv,k} + v_{t,bk} \cdot \ln \left( \frac{i_{st}}{i_{o,bk}} + 1 \right) \quad (4)$$

Instead, the model proposed in (Petrone & Ramos, 2011) does not require the Lambert-W function: the same equations system in (3)-(4) must be solved, but since  $R_h$  and  $R_s$  are not considered, each component of (3) is an explicit equation. Moreover, since the bypass diode is modeled by an ideal switch, the number of equations in (3) changes inversely with the number of bypass diodes active. Therefore, the Inflection Points model requires a significantly shorter simulation time in comparison with the Lambert-W approach, but it introduces calculation errors due to the simplifications adopted. In any case, the model in (Petrone & Ramos, 2011) still requires to solve a non-linear equation system, which uses an optimization algorithm such as the Newton-Raphson or trust-region methods.

The algorithm proposed in this paper overcomes the main

drawbacks of the previous solutions. The algorithm is divided in three parts as depicted in Fig. 5: the first process is used to obtain the I-V curve of each PV module independently, it using (1) and (2), which are explicitly evaluated for different values of  $v_d$ . Such an approach avoids the requirement of the Lambert-W function without introducing simplification errors. The second procedure is used to obtain the string I-V curve by adding the modules voltage at the evaluated string current. Since all the modules are in series their current is the same, hence the modules voltages are explicitly extracted from the modules I-V curves generated by the first procedure. Then, both processes are iteratively executed for each irradiance value to estimate the power and energy production. Finally, the proposed approach does not require to solve a non-linear equation system, hence it provides high accuracy and short simulation times.

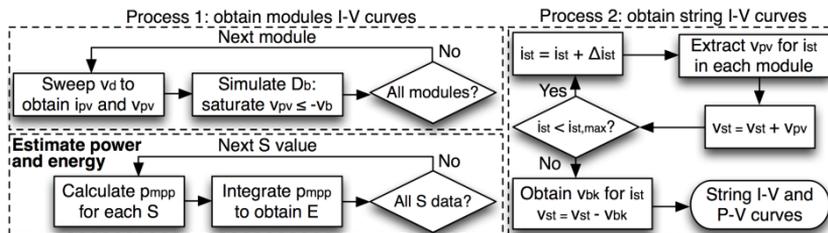


Fig. 5. Proposed approach to estimate the string power and energy production. Source: Authors

#### 4. PERFORMANCE EVALUATION OF THE PROPOSED APPROACH

The performance of the proposed approach is evaluated in contrast with both the accurate (Lambert-W) and the fast (Inflection points) solutions. Fig. 6 shows the simulation of a string with  $N = 5$ , an ambient irradiance of  $1000 \text{ W/m}^2$  and the presence of several shade sources that generate the following shading profile: the module PV1 receives the 94 % of the irradiance, while the modules PV2 to PV5 receive 60 %, 40 %, 20 % and 10 %, respectively. The figure illustrates the accurate simulation performed by the proposed approach, which results are indistinguishable from the Lambert-W simulation, while the Inflection Points approach in-

roduces significant errors. To provide a more general comparison, Fig. 7 presents the simulation times  $T_p$  and normalized errors NSSE (Saavedra et al., 2011) for PV string sizes between 2 and 10 modules.

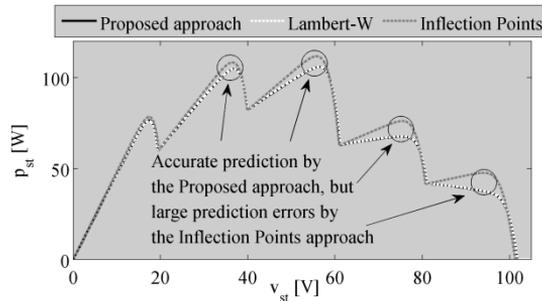


Fig. 6. Simulation of string with  $N = 5$ . Source: Authors

The results put in evidence the strong reduction in the simulation time provided by the proposed approach: in Fig. 7 the  $T_p$  of the Lambert-W approach is divided by 100 to allow its plotting in the same scale of the Inflection points and proposed approaches. In example, for  $N = 6$ , the proposed approach requires 0.034 s, while the Inflection Points and Lambert-W require 1.40 s and 191.95 s, respectively. Similarly, the error generated by the proposed approach, with respect to the more accurate Lambert-W, is much smaller than the error introduced by the Inflection Points solution. In any case, the error generated by the proposed approach is negligible: in example, for  $N = 6$ , the proposed approach introduces an error of 0.12 %, while the Inflection Points solution introduces an error of 2.88 %.

In addition, the results show that for larger  $N$  the difference between the simulation times increases, but the errors are almost constant. Hence, the proposed approach provides accurate results in very short times, which allows to estimate the power production of large PV strings in long periods.

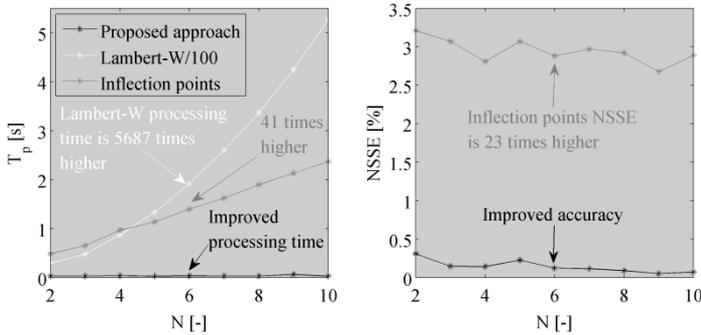


Fig. 7. Simulation accuracy and processing times for multiple string sizes. Source: Authors

### 5. APPLICATION EXAMPLE

To illustrate the use of the proposed solution, a realistic application is considered: for a commercial PV inverter (SolarEdge SE2200) that requires 350 V to operate, 20 BP-585 PV modules must be used to form the string. But, considering several shade sources in the available area that produce the profile described in Table 1, it is required to estimate the yearly energy production to evaluate the economic convenience of the PV installation.

Table 1. Shading profile for the application example. Source: Authors

PV1-PV10	94%	86%	66%	61%	44%	42%	41%	40%	39%	38%
PV11-PV20	37%	33%	31%	28%	27%	26%	20%	10%	5%	4%

The simulation of a single irradiance condition takes 1380 s (23 minutes) with the Lambert-W approach, 6.5 s with the Inflection Points approach, while the proposed approach requires 0.0061 s with an error of 0.1 %. To simulate a day (8.5 hours with data each 30 s), the Lambert-W will require 391 hours (16.3 days), the Inflection Points will require 1.84 hours, while the proposed approach requires 6.24 s. Finally, to simulate a year (365 days), the Lambert-W will require 16.4 years, the Inflection Points will require 28 days, while the proposed approach requires 37 minutes.

Therefore, the yearly simulation of the string with N = 20 is

not practical with both the Lambert-W or Inflection Points approaches. Instead, the proposed approach makes possible to perform the evaluation: Fig. 8 shows the simulation of a day in southern Italy performed in 6.24 s, where the irradiance profile is presented at the left and the power production at the right. Similarly, Fig. 9 shows the yearly energy production, where the irradiance profile in Fig. 8 changes depending on the month. Such a simulation was performed in 37 minutes. Finally, with the information provided in Fig. 9 it is possible to calculate the return-of-investment time (in years) using the costs of the PV system and kWh in the geographical region under evaluation.

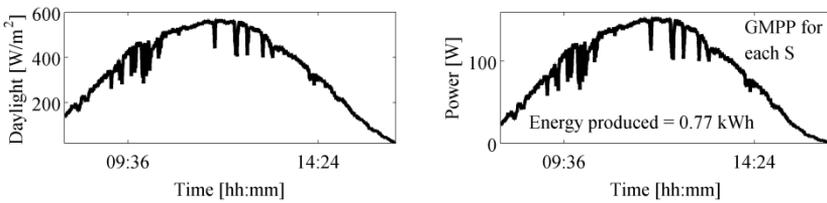


Fig. 8. Simulation of day for  $N = 20$ , performed in 6.24 s. Source: Authors

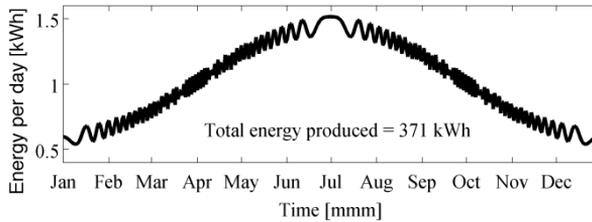


Fig. 9. Simulation of a year for  $N = 20$ , performed in 37 min. Source: Authors

## 6. CONCLUSIONS

A novel approach to estimate the power and energy production in urban PV strings was proposed. The method is based on an accurate PV cell model and an approximated diode model, but introducing an efficient algorithm to simulate the string without requiring the Lambert-W function or to solving a non-linear equation system. Instead,  $N$  non-nested and explicit equations are used to calculate the string power, which eventually provides high

accuracy and low processing times. The results put in evidence the usefulness of such an approach to simulate long periods with high precision. Finally, the method could be extended to consider multiple strings to evaluate PV arrays.

## 7. ACKNOWLEDGMENT

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